APPLICATION NOTE

DID YOU KNOW ?
Gustav Robert Kirchhoff (1824-1887) the German Physicist who gave us "Kirchhoff's voltage (current) law" invented the Bunsen Burner working together with Robert Wilhelm Bunsen, a German Chemist.

Measuring RMS Values of Voltage and Current

Voltage (Current) Measurements

Standard classic measurements of voltage (current) values are based on two fundamental techniques either "average" or "effective".

The "average" value of a function of time is the net area of the function calculated over a specific interval of time divided by that time interval.

Specifically,

$$Vavg = \left(\frac{1}{T2 - T1}\right) * \int_{T1}^{T2} V(t)dt \qquad \textbf{Eqn 1}$$

If a voltage (current) is either constant or periodic, then measuring its average is independent of the interval over which a measurement is made. If, on the other hand, the voltage (current) function grows without bound over time, the average value is dependent on the measurement interval and will not necessarily be constant, i.e. no average value exists. Fortunately in the practical electrical world values of voltage (current) do not grow in a boundless manner and, therefore, have well behaved averages. This is a result of the fact that real voltage (current) sources are generally either; (1) batteries with constant or slowly (exponentially) decaying values, (2) bounded sinusoidal functions of time, or (3) combinations of the above. Constant amplitude sinusoidal functions have a net zero average over time intervals, which are equal to integer multiples of the sinusoidal period. Moreover, averages can be calculated over an infinite number of intervals, which are not equal to the sinusoidal period. These averages are also zero. Although the average of a bounded sinusoidal function is zero, the "effective" value is not zero. For example, electric hot water heaters work very well on sinusoidal voltages, with zero average values.

Effective Value

The "effective" value of symmetrical periodic voltage (current) functions of time is based on the concept of "heating capability". Consider the test fixture shown in Figure 1.

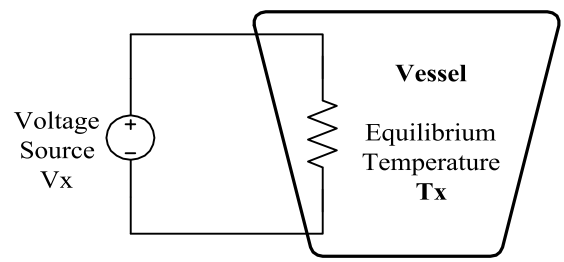


Figure 1
Test Fixture

This vessel is insulated and filled with some stable liquid (transformer oil for example) capable of reaching thermodynamic equilibrium. If a DC voltage Vx is applied to the vessel's internal heater, the liquid temperature will rise. Eventually, the electrical energy applied to this vessel will establish an equilibrium condition where energy input equals energy (heat) lost and the vessel liquid will arrive at an equilibrium temperature, Tx degrees.

Next in this experimental scenario, replace the DC voltage source Vx with a time varying voltage which does not increase without bound. Eventually, in some time T_{final} , thermal equilibrium will again be established. If this equilibrium condition establishes the same temperature Tx as reached before with the applied DC voltage Vx, then one can say that the "effective" value of this time varying function is Vx.

Hence the definition of "effective value".

Equation 2 illustrates this thermal equilibrium.

$$((V_{Effective})^2 / R) * T_{final} = \int_0^{T_{final}} (V(t)^2 / R)dt \qquad \textbf{Eqn 2}$$

If V(t) is a periodic function of time with a cycle period of Tp, and T_{final} is an integer "n" times the period (n*Tp) then the integral over T_{final} is simply n times the integral over Tp. The results of these substitutions are shown in Equation 3.

$$V_{Effective} = \sqrt{(1/Tp) * \int_0^{Tp} V(t)^2 dt}\,, \quad \textbf{RMS} \qquad \textbf{Eqn 3}$$

Equation 3 illustrates that the effective equivalent heating capacity of a bounded periodic voltage (current) function can be determined over just one cycle. This equation is recognized as the old familiar form of "*square **R**oot of the **M**ean (average) **S**quared*"; hence, the name, "RMS".

Examples of Using the "RMS" Equation

The following results can be shown by direct application of Eqn 3.

1. Sinusoidal function, peak of Vp

   $VRMS = Vp \div \sqrt{2}$; Vp*0.707

2. Symmetrical Periodic Pulse Wave, peak of Vp

   VRMS = Vp (Symmetric Square Wave)

3. Non-symmetrical Periodic Pulse Wave, all positive peaks of Vp, with duty cycle D

   $VRMS = Vp * \sqrt{D}$

   D ≡ Td/Tp, Pulse duration Td ÷ Period Tp

4. Symmetrical Periodic Triangle Wave, peak Vp

   $VRMS = Vp \div \sqrt{3}$; Vp * 0.5774 (Saw-Tooth)

5. Full wave Rectified Sinusoid, peak Vp

   $VRMS = Vp \div \sqrt{2}$; Vp * 0.707

6. Half Wave Rectified Sinusoid, peak Vp

   VRMS = Vp ÷2; Vp * 0.5

Note: These examples illustrate that the shape of a periodic function can determine its RMS value. The peak (crest) of a voltage (current) function of time divided by $\sqrt{2}$ is often mistakenly used to calculate the RMS value. This technique can result in errors and clearly should be avoided.

Effective (RMS) Values of Complex Functions

An extremely useful fact in determining RMS values is that any well behaved bounded periodic function of time can be expressed as an average value plus a sum of sinusoids (Fourier's Theorem), for example;

$$V(t) = Ao + \sum [\, An*Cos(n\omega_o t) + Bn*Sin(n\omega_o t)\,]$$

Summed over all "n" values **Eqn 4**

Where ω_o is the radian frequency of V(t) and An, Bn, Ao are Fourier Amplitude Coefficients.

When this series is substituted in the integral expression Equation 2 for RMS, one obtains the following;

$$Vrms = \sqrt{\{\sum[(A_0)^2 + (A_n)^2/2 + (B_n)^2/2]\}}$$

Summed over all "n" values **Eqn 5**

Note: $(A_n)^2/2$ and $(B_n)^2/2$ are the squares of RMS values for each n^{th} Sin and Cosine component.

The important conclusion is;

A bounded periodic function of* time *has a RMS value equal to the square root of the sum of the square of each individual component's RMS value.

Practical Considerations

Figure 2 illustrates composite curves formed by adding two sinusoids, one at 60 Hz and one at 180Hz. Curve 1 is for zero phase difference and Curve 2 is for a 90-degree phase difference.

Specifically;

Curve 1 V(t) = 170*Sin(377*t) +50*Sin(1131*t)
Curve 2 V(t) = 170*Sin(377*t) +50*Cos(1131*t)

Note: Composite curve shape is determined by phase and frequency harmonics.

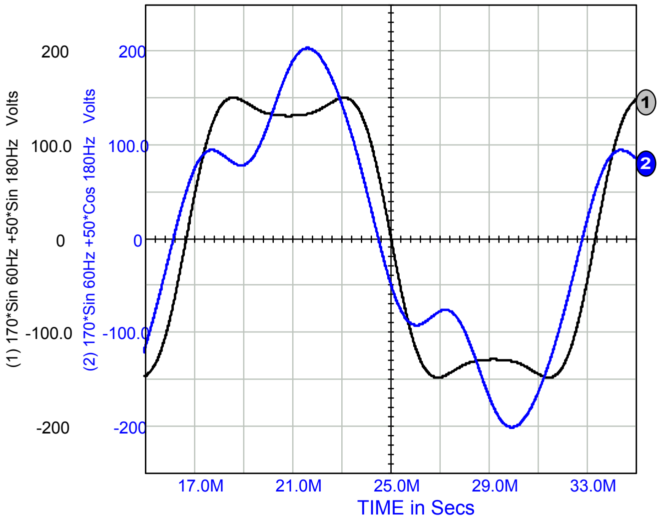


Figure 2
Fundamental with Third Harmonic Added
Curve 2 170*Sin(377*t) +50*Cos(1131*t)
Curve 1 170*Sin(377*t) +50*Sin(1131*t)

Industrial sinusoidal functions of voltage (current) often contain harmonics that impact wave shape and peak (crest) values. For example, Curve 2 is typical of the magnetizing currents in 60 Hz transformers and motors. Inexpensive RMS reading devices often use a rectifier circuits that capture the peak value, which is then scaled by 0.707 and displayed as RMS. Clearly this technique can give incorrect RMS readings. In this example, using Vpeak $\div\sqrt{2}$ clearly gives incorrect values.

Curve 1: 203*0.707 = 144 volts, not true RMS
Curve 2: 155*0.707 = 110 volts, not true RMS

The correct RMS value for both of these composite sinusoidal functions is;

$[\,(170)^2/2 + (50)^2/2\,]^{1/2}$ = 125.3 volts RMS

Table 1 illustrates two examples of RMS calculations by using individual Fourier coefficients and Eqn 5. Example one is a full wave rectified 1-volt peak sinusoid. Note that for a full wave rectified function the measurement device needed to achieve a RMS reading within 0.01% error requires a bandwidth, which includes the fifth (5) harmonic and the resolution to read 10 mV levels.

The other example illustrated in Table 1 is a saw-tooth 1-volt peak function. For this example, the measurement device for a saw-tooth function needed to achieve an RMS reading within 0.3% error requires a bandwidth, which includes the twenty-fifth (25) harmonic and the resolution to read 10 mV levels.

Assume, for illustration purposes, that an AC ripple on the DC output of a rectifier can be approximated by a saw-tooth function. Table 1 illustrates that to measure within a 0.3% error the AC RMS ripple on the DC output of a 20 kHz rectifier the measurement device must have a bandwidth in excess of 500 kHz and a resolution to read voltage levels down by 40 dB (100 microvolts for a peak 10 mV ripple). This example clearly illustrates that signal shape, together with the measurement bandwidth and resolution are extremely important in determining the accuracy of measuring true RMS.

Any "true RMS" measurement device must be capable of accurately implement Eqn 3. The subtlety in this statement is that electronically implementing Eqn 3, requires a device to have a very large bandwidth and be able to resolve small magnitudes.

Crest Factor

Another figure of merit often used to characterize a periodic time function of voltage (current) is the Crest Factor (CF). The Crest Factor for a specific waveform is defined as the peak value divided by the RMS value. Specifically,

$CF = V_{peak} / V_{RMS}$ **Eqn 6**

Examples: (from page 2)

1. Pure Sinusoid, $CF = \sqrt{2}$
2. Symmetrical Periodic Pulses, CF = 1
3. Non-symmetrical Periodic Pulses with duty cycle D, $CF = 1 \div \sqrt{D}$

   Example; If D = 5%, CF = 4.47
4. Symmetrical Periodic Triangle, $CF = \sqrt{3}$
5. Full wave Rectified Sinusoid, $CF = \sqrt{2}$
6. Half Wave Rectified Sinusoid, CF = 2

From Figure 2;

Curve 1, CF = 1.62

Curve 2, CF = 1.24

DATAFORTH RMS MEASUREMENT DEVICES

True RMS measurements require instrumentation devices that accurately implement Eqn 3, "the" RMS equation. These devices must have both wide bandwidths and good low level resolution to support high Crest Factors. **Dataforth** has developed two products that satisfy these requirements; the SCM5B33 and DSCA33 True RMS Input modules. Both these products provide a 1500Vrms isolation barrier between input and output. The SCM5B33 is a plug-in-panel module, and the DSCA33 is a DIN rail mount device. Each provide a single channel of AC input that is converted to its True RMS DC value, filtered, isolated, amplified, and converted to standard process voltage or current output.

SCM5B33 ISOLATED TRUE RMS INPUT MODULE, PLUG-IN-PANEL MOUNT

FEATURES

- INTERFACES RMS VOLTAGE (0 - 300V) OR RMS CURRENT (0 - 5A)
- DESIGNED FOR STANDARD OPERATION WITH FREQUENCIES OF 45HZ TO 1000HZ (EXTENDED RANGE TO 20Khz)
- COMPATIBLE WITH STANDARD CURRENT AND POTENTIAL TRANSFORMERS
- INDUSTRY STANDARD OUTPUTS OF EITHER 0-1MA, 0-20ma, 4-20 MA, 0-5V OR 0-10VDC
- ±0.25% FACTORY CALIBRATED ACCURACY (ACCURACY CLASS 0.2)
- 1500 VRMS CONTINUOUS TRANSFORMER BASED ISOLATION
- INPUT OVERLOAD PROTECTED TO 480V MAX (PEAK AC & DC) OR 10A RMS CONTINUOUS
- ANSI/IEEE C37.90.1-1989 TRANSIENT PROTECTION

  CSA AND FM APPROVALS PENDING

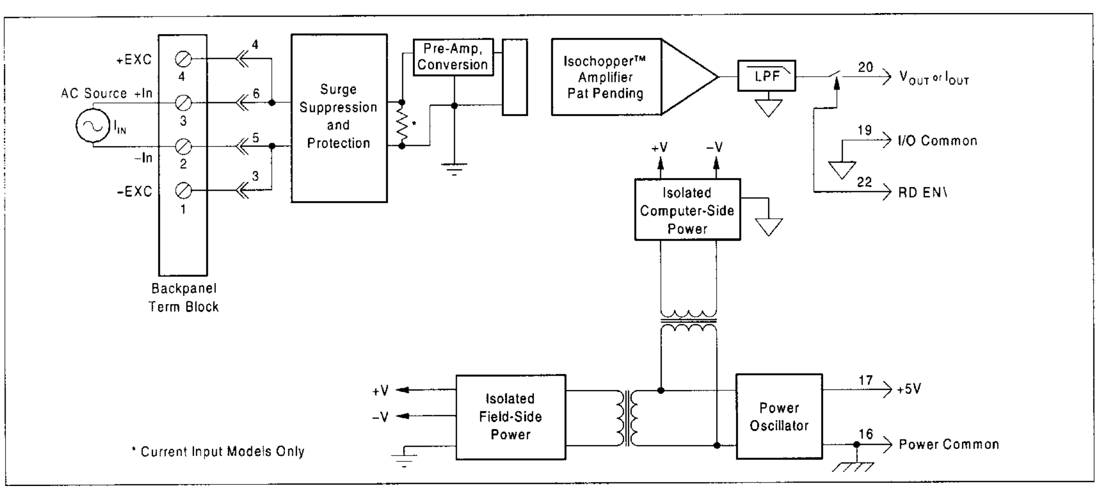
DESCRIPTION

Each SCM5B33 True RMS input module provides a single channel of AC input which is converted to its True RMS dc value, filtered, isolated, amplified, and converted to a standard process voltage or current output (see diagram below).

The SCM5B modules are designed with a completely isolated computer side circuit, which can be floated to ±50V from Power Common, pin 16. This complete isolation means that no connection is required between I/O Common and Power Common for proper operation of the output switch. If desired, the output switch can be turned on continuously by simply connecting pin 22, the Read-Enable pin to I/O Common, pin 19.

The field voltage or current input signal is processed through a pre-amplifier and RMS converter on the field side of the isolation barrier. The converted dc signal is then chopped by a proprietary chopper circuit and transferred across the transformer isolation barrier, suppressing transmission of common mode spikes and surges. The computer side circuitry reconstructs filters and converts the signal to industry standard outputs. Modules are powered from +5VDC, ±5%.

For current output models an external loop supply is required having a compliance voltage of 14 to 48VDC. Connection, with series load, is between Pin 20 (+) and Pin 19 (-).



DSCA33 ISOLATED TRUE RMS INPUT MODULE, DIN RAIL MOUNT

FEATURES

- INTERFACES RMS VOLTAGE (0 - 300V) OR RMS CURRENT (0 - 5A)
- DESIGNED FOR STANDARD OPERATION WITH FREQUENCIES OF 45HZ TO 1000HZ (EXTENDED RANGE OPERATION TO 20kHZ)
- COMPATABLE WITH STANDARD CURRENT AND POTENTIAL TRANSFORMERS
- INDUSTRY STANDARD OUTPUTS OF EITHER 0-1MA, 0-20MA, 4-20MA, 0-5V, OR 0-10VDC
- ±0.25% FACTORY CALIBRATED ACCURACY (ACCURACY CLASS 0.2)
- ±5% ADJUSTABLE ZERO AND SPAN

  1500 VRMS CONTINUOUS TRANSFORMER BASED ISOLATION
- INPUT OVERLOAD PROTECTED TO 480V (PEAK AC & DC) OR 10A RMS CONTINUOUS
- ANSI/IEEE C37.90.1-1989 TRANSIENT PROTECTION
- MOUNTS ON STANDARD DIN RAIL
- CSA AND FM APPROVALS PENDING

DESCRIPTION

Each DSCA33 True RMS input module provides a single channel of AC input which is converted to its True RMS DC value, filtered, isolated, amplified, and converted to standard process voltage or current output (see diagram below).

The field voltage or current input signal is processed through an AC coupled pre-amplifier and RMS converter on the field side of the isolation barrier. The converted DC signal is then filtered and chopped by a proprietary chopper circuit and transferred across the transformer isolation barrier, suppressing transmission of common mode spikes and surges.

Module output is either voltage or current. For current output models a dedicated loop supply is provided at terminal 3 (+OUT) with loop return located at terminal 4 (-OUT).

Special input circuits provide protection against accidental connection of power-line voltages up to 480VAC and against transient events as defined by ANSI/IEEE C37.90.1-1989. Protection circuits are also present on the signal output and power input terminals to guard against transient events and power reversal. Signal and power lines are secured to the module using pluggable terminal blocks.

DSCA33 modules have excellent stability over time and do not require recalibration, however, both zero and span settings are adjustable to accommodate situations where fine-tuning is desired. The adjustments are made using potentiometers located under the front panel label and are non-interactive for ease of use.

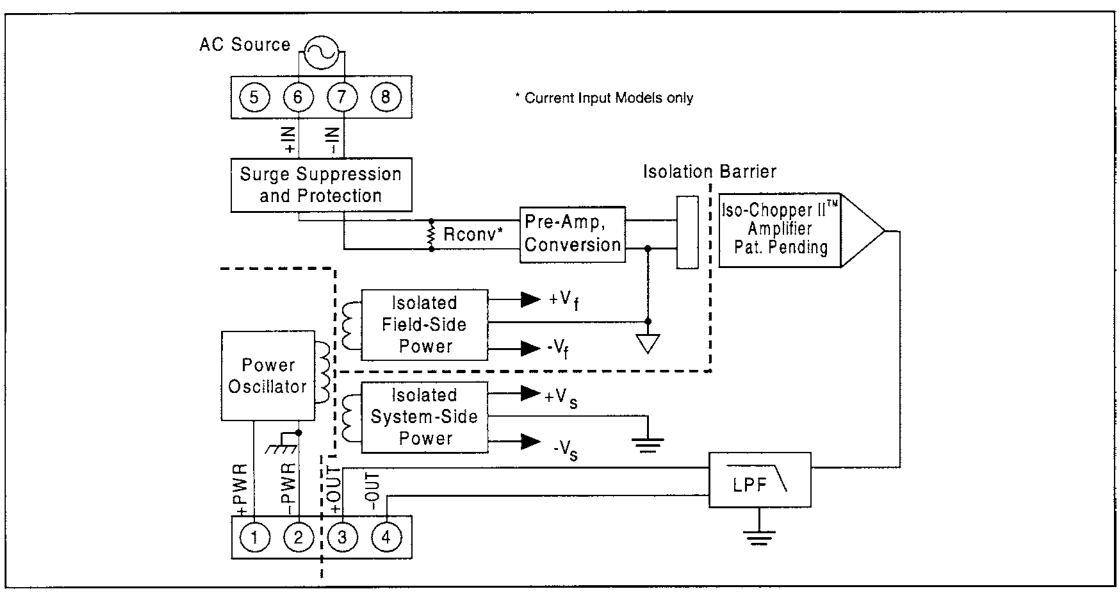


Table 1

RMS Calculated from Individual Fourier Coefficients

	Full-wave Rectified,1 Volt Peak				Saw-Tooth Function,1 Volt Peak			
n	An	An (rms^2)	Total rms	% Error	Bn	Bn (rms^2)	Total rms	% Error
0	6.36620E-01	4.05285E-01	6.366198E-01	9.9684	5.00000E-01	2.500E-01	5.0000E-01	13.397
1	4.24413E-01	9.00633E-02	7.038096E-01	0.4663	3.18310E-01	5.066E-02	5.4833E-01	5.027
2	8.48826E-02	3.60253E-03	7.063643E-01	0.1050	1.59155E-01	1.267E-02	5.5976E-01	3.048
3	3.63783E-02	6.61689E-04	7.068325E-01	0.0388	1.06103E-01	5.629E-03	5.6476E-01	2.181
4	2.02102E-02	2.04225E-04	7.069770E-01	0.0184	7.95775E-02	3.166E-03	5.6756E-01	1.696
5	**1.28610E-02**	8.27027E-05	**7.070355E-01**	**0.0101**	6.36620E-02	2.026E-03	5.6934E-01	1.388
6	8.90377E-03	3.96386E-05	7.070635E-01	0.0061	5.30516E-02	1.407E-03	5.7057E-01	1.174
7	6.52943E-03	2.13168E-05	7.070786E-01	0.0040	4.54728E-02	1.034E-03	5.7148E-01	1.017
8	4.99310E-03	1.24655E-05	7.070874E-01	0.0027	3.97887E-02	7.916E-04	5.7217E-01	0.897
9	3.94192E-03	7.76936E-06	7.070929E-01	0.0020	3.53678E-02	6.254E-04	5.7272E-01	0.802
10	3.19108E-03	5.09148E-06	7.070965E-01	0.0015	3.18310E-02	5.066E-04	5.7316E-01	0.726
11	2.63611E-03	3.47453E-06	7.070989E-01	0.0011	2.89373E-02	4.187E-04	5.7352E-01	0.663
12	2.21433E-03	2.45163E-06	7.071007E-01	0.0009	2.65258E-02	3.518E-04	5.7383E-01	0.609
13	1.88628E-03	1.77903E-06	7.071019E-01	0.0007	2.44854E-02	2.998E-04	5.7409E-01	0.564
14	1.62610E-03	1.32211E-06	7.071029E-01	0.0006	2.27364E-02	2.585E-04	5.7432E-01	0.525
15	1.41628E-03	1.00293E-06	7.071036E-01	0.0005	2.12207E-02	2.252E-04	5.7451E-01	0.491
16	1.24461E-03	7.74531E-07	7.071041E-01	0.0004	1.98944E-02	1.979E-04	5.7469E-01	0.461
17	*	*	*	*	1.87241E-02	1.753E-04	5.7484E-01	0.435
18	*	*	*	*	1.76839E-02	1.564E-04	5.7497E-01	0.412
19	*	*	*	*	1.67532E-02	1.403E-04	5.7510E-01	0.390
20	*	*	*	*	1.59155E-02	1.267E-04	5.7521E-01	0.371
21	*	*	*	*	1.51576E-02	1.149E-04	5.7531E-01	0.354
22	*	*	*	*	1.44686E-02	1.047E-04	5.7540E-01	0.338
23	*	*	*	*	1.38396E-02	9.577E-05	5.7548E-01	0.324
24	*	*	*	*	1.32629E-02	8.795E-05	5.7556E-01	0.311
25	*	*	*	*	**1.27324E-02**	8.106E-05	**5.7563E-01**	**0.298**
		Exact RMS	**7.071068E-01**	**0**		**Exact RMS**	**5.7735E-01**	**0**